Measuring and detecting quantum entanglement or nonlocality via Hong-Ou-Mandel interference

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Detecting entanglement

Two-qubit density matrix ρ (15 real parameters)

in Hilbert-Schmidt form

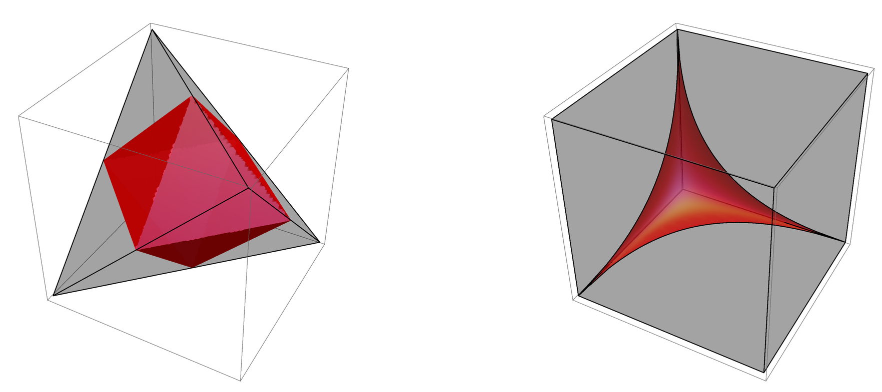
$$\rho = \frac{1}{4}(I \otimes I + \vec{x} \cdot \vec{\sigma} \otimes I + I \otimes \vec{y} \cdot \vec{\sigma} + \sum_{i,j=1}^{3} T_{i,j}\, \sigma_i \otimes \sigma_j),$$

where $\vec{\sigma} = [\sigma_1, \sigma_2, \sigma_3]$. The elements of the Bloch vectors are $x_i = \mathrm{Tr}[\rho(\sigma_i \otimes I)]$ and $y_i = \mathrm{Tr}[\rho(I \otimes \sigma_i)]$, respectively.

$$T_{i,j} = \mathrm{Tr}[\rho(\sigma_i \otimes \sigma_j)]$$

Can be diagonalized with only local operations (HWPs, QWPs).

Geometric pictures



Space of matrix T (9 real parameters)

Each point is given as

$$\vec{t} = [T_{1,1}, T_{2,2}, T_{33}],$$

where T is a diagonal matrix. All physical states can be moved to the tetrahedron $\mathcal{T}$ (or $-\mathcal{T}$). All separable states are found in the octahedron [1].

Space of matrix R (6 real parameters)

It is convenient to use real, positive and symmetric matrix $R = T^T T$ (6 real parameters). In this representation each physical state is now found in a cube.

Two-copy formula

$$R_{i,j} = \mathrm{Tr}\left[(\rho_{a_1b_1} \otimes \rho_{a_2b_2}) S_{a_1a_2} \otimes (\sigma_i \otimes \sigma_j)_{b_1b_2}\right],$$

where $S_{a_1a_2} = (I - 4|\Psi^-\rangle\langle\Psi^-|)_{a_1a_2}$ (HOM interference) [2]

Fully entangled fraction

Fidelity

of teleportation based technologies

$$f = \frac{1}{6}(|t_1| + |t_2| + |t_3| + 2)$$

or $f = \frac{1}{3}(1 + 2\mathrm{FEF}) = \frac{1}{6}(\mathrm{Tr}\sqrt{R} + 2)$. This is related to the probability of successful teleportation, entanglement swapping, generating a secure cryptographic key bit etc.

FEF quantifier

$$F = \frac{1}{2}(\mathrm{Tr}\sqrt{R} - 1)$$

Entropic entanglement witness

Purity (linear entropy)

Purity of separable state (lin. entropy $\propto 1 - \mathrm{Tr}\rho^2$)

$$\mathrm{Tr}\rho_{ab}^2 \geq \min(\mathrm{Tr}\rho_a^2, \mathrm{Tr}\rho_b^2)$$

Entropic witness

$$E = 2[\mathrm{Tr}\rho_{ab}^2 - \min(\mathrm{Tr}\rho_a^2, \mathrm{Tr}\rho_b^2)] = \frac{1}{2}(\mathrm{Tr}R - 1 + |\mathrm{Tr}\rho_a^2 - \mathrm{Tr}\rho_b^2|)$$

Bell CHSH nonlocality

Bell CHSH inequality

For local states

$$\max_{\mathcal{B}_{\mathrm{CHSH}}} |\mathrm{Tr}(\rho\, \mathcal{B}_{\mathrm{CHSH}})| = 2\sqrt{\mathrm{Tr}R - \min[\mathrm{eig}(R)]} \leq 2,$$

where $\mathcal{B}_{\mathrm{CHSH}} = \hat{a} \cdot \vec{\sigma} \otimes (\hat{b} + \hat{b}') \cdot \vec{\sigma} + a' \cdot \vec{\sigma} \otimes (\hat{b} - \hat{b}') \cdot \vec{\sigma}$ depends on unit vectors in 3D real space $\hat{a}, \hat{b}, \hat{a}', \hat{b}'$ [5]. For correlations that do not break Bell's (CHSH) inequality (insecure QKD) one can construct a local hidden variable model [4, 3].

CHSH nonlocality measure

$$M = \mathrm{Tr}R - \min[\mathrm{eig}(R)] - 1$$

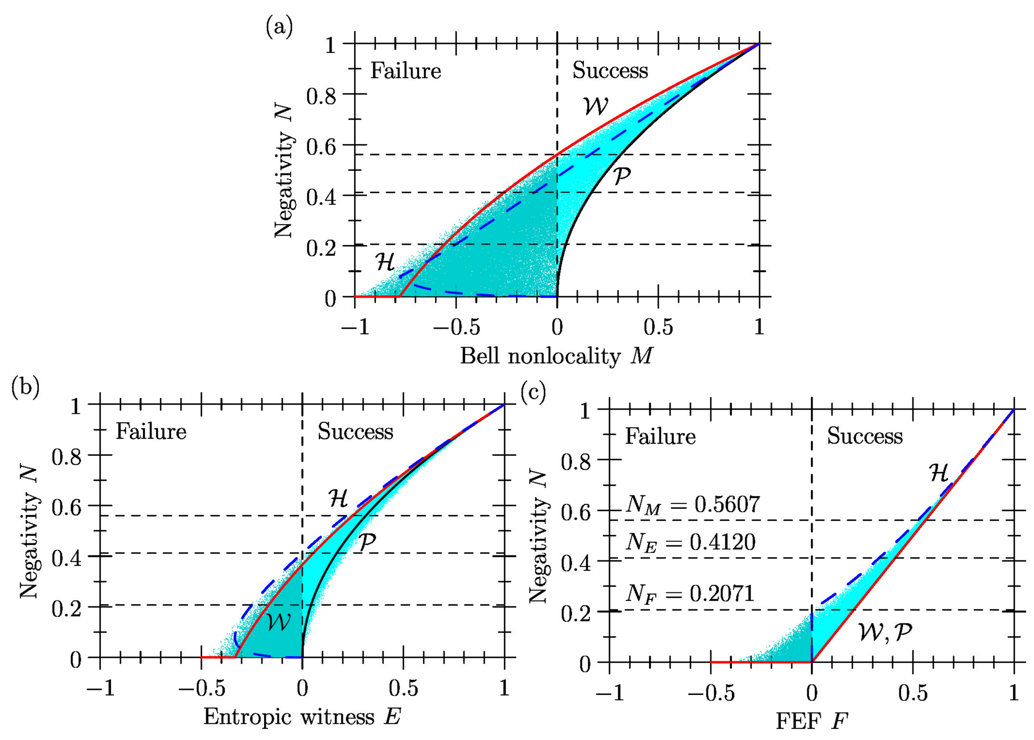
Measuring entanglement

Negativity

$$N(\rho) = \max\{-2\min[\mathrm{eig}(\rho^\Gamma), 0\},$$

Γ – partial transposition [6]

The entanglement cost under the PPT operations is $\log_2(N + 1)$ [7].



Characteristic two-qubit states

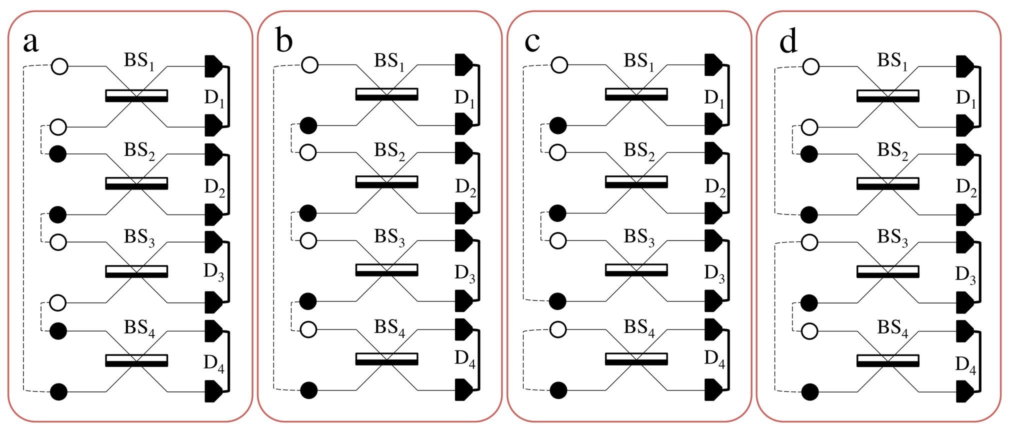
$\mathcal{W} = \frac{q}{4}I + p|\Psi^-\rangle\langle\Psi^-|$, $\mathcal{H} = p|HH\rangle\langle HH| + q|\Psi^-\rangle\langle\Psi^-|$,

$\mathcal{P} = (\sqrt{p}|HH\rangle + \sqrt{q}|VV\rangle)(\mathrm{H.c.})$, for $q = 1 - p$ and $0 \leq p \leq 1$

Interferometers

Four-copy HOM interference

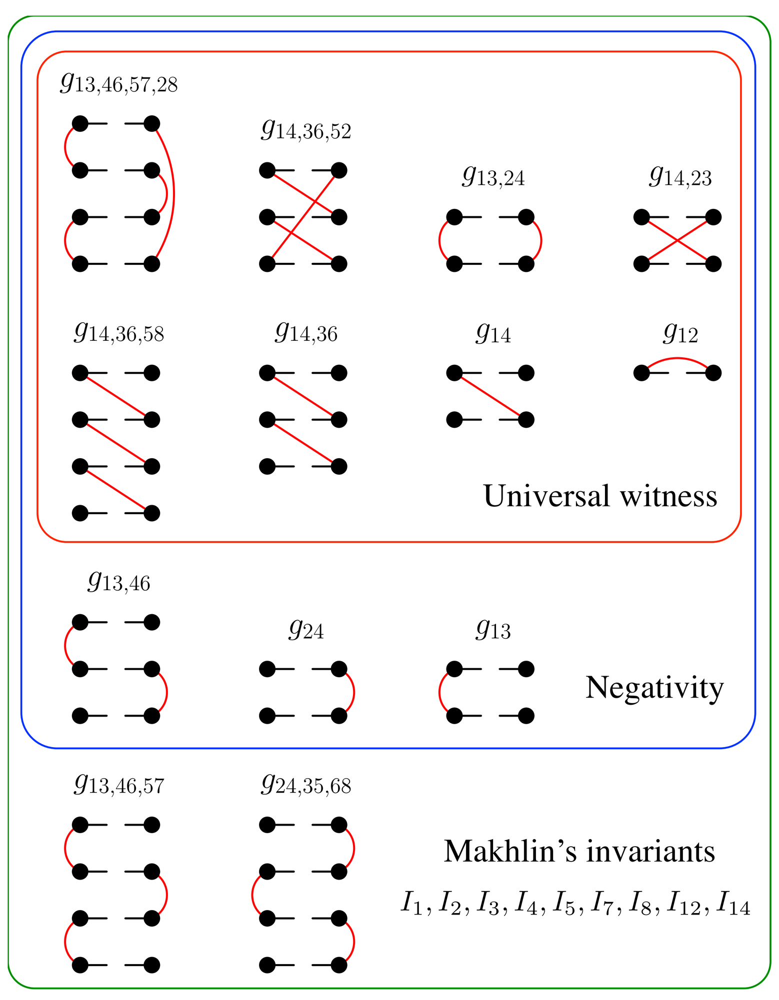
There are 2 interferometric events that do not matter for measuring N [8].



Web of singlet projections

Prime detection events

Events are found with local invariants and Cayley-Hamilton theorem [8].



Acknowledgments

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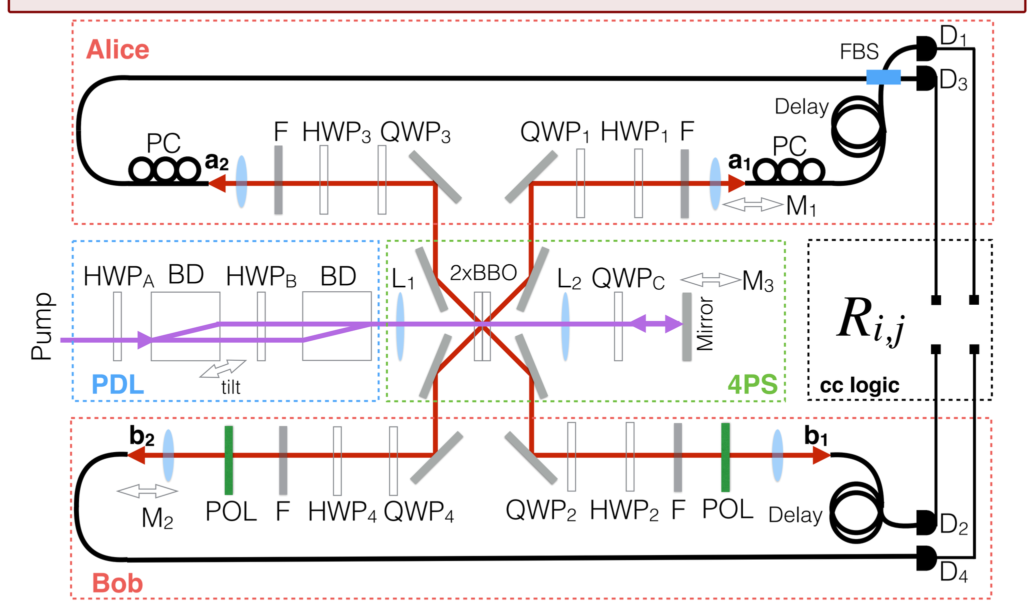
Two-copy experiment

Alice

- Measurement: $(\frac{r}{2}I + (1-r)|\Psi^-\rangle\langle\Psi^-|)_{a_1a_2}$
- r - tunable fraction of noninterfering photons



Bob

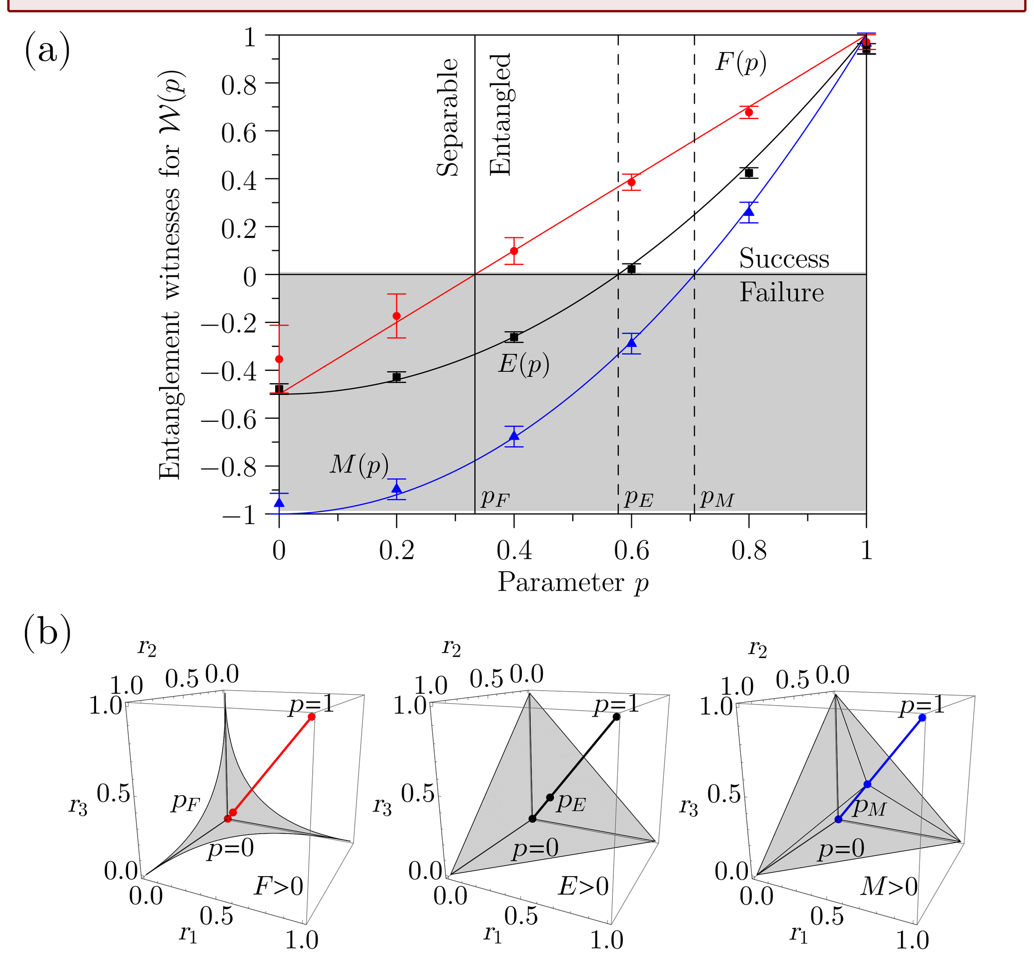
Measurement: $(\sigma_i \otimes \sigma_j)_{b_1b_2}$

This experimental setup was used in Refs. [9, 10].

Experimental results

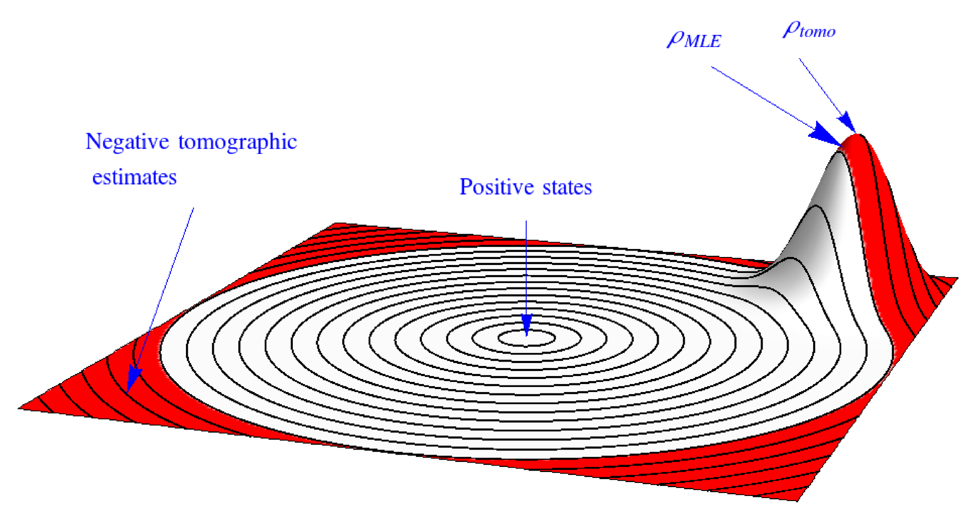
Experimental results for the Werner states [11]

$\mathcal{W} = \frac{1-p}{4}I + p|\Psi^-\rangle\langle\Psi^-|$ $p_F = \frac{1}{3}$, $p_E = \frac{1}{\sqrt{3}}$, $p_M = \frac{1}{\sqrt{2}}$.



Maximum likelihood estimation (MLE)

To ensure the positivity of the matrices, we use the maximum-likelihood method developed for quantum state tomography [12].



MLE for R matrix

All physical R matrices are found in the cube, where the probabilities of coincidence detection are properly defined for any measurement basis. The most likely physical matrix R, for which $0 \leq r_j \leq 1$ for $j = 1, 2, 3$ and $[r_1, r_2, r_3] = \mathrm{eig}(R)$, is found by maximizing the logarithmic likelihood function

$$\mathcal{L} = -\sum_{1 \leq i \leq j}^{3} \left(\frac{R_{i,j}^{(\mathrm{exp})} - R_{i,j}}{\delta R_{i,j}^{(\mathrm{exp})}}\right)^2$$

for experimental results $R^{(\mathrm{exp})}$ and their uncertainties $\delta R^{(\mathrm{exp})}$.

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