Experimental measurement of Hilbert-Schmidt distance between two-qubit states

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Hilbert-Schmidt distance between two-qubit states

The measures of distances between points in a Hilbert space are one of the basic theoretical concepts used to characterize properties of a quantum system with respect to some etalon state. These are not only used in studying fidelity of signal transmission and basic quantum phenomena but also applied in measuring quantum correlations, and also in quantum machine learning. The values of quantum distance measures are very difficult to determine without completely reconstructing the state.

Here, we focus on a direct measurement of Hilbert-Schmidt distance between two-qubit states [1].

$$H(\rho_1, \rho_2) = \sqrt{\mathrm{Tr}(\rho_1 - \rho_2)^2}$$

The method for direct measurement of the HSD requires simultaneous interaction between four qubits. That is, for instance, four photons using only one degree of freedom (DOF) such as polarization. In our setup, however, we have used two DOFs (polarization and spatial) to encode qubits, therefore, only two photons were needed. This way one achieves much higher detection rates which makes the experiment considerably faster.

Experimental setup and measurement

The two photons are generated in a crystal cascade (known as the Kwiat source [2]) pumped by pulsed Paladine (Coherent) laser at λ = 355 nm with 300 mW of mean optical power and repetition rate of 120 MHz. The source generates polarization–entangled photon pairs at λ = 710 nm.

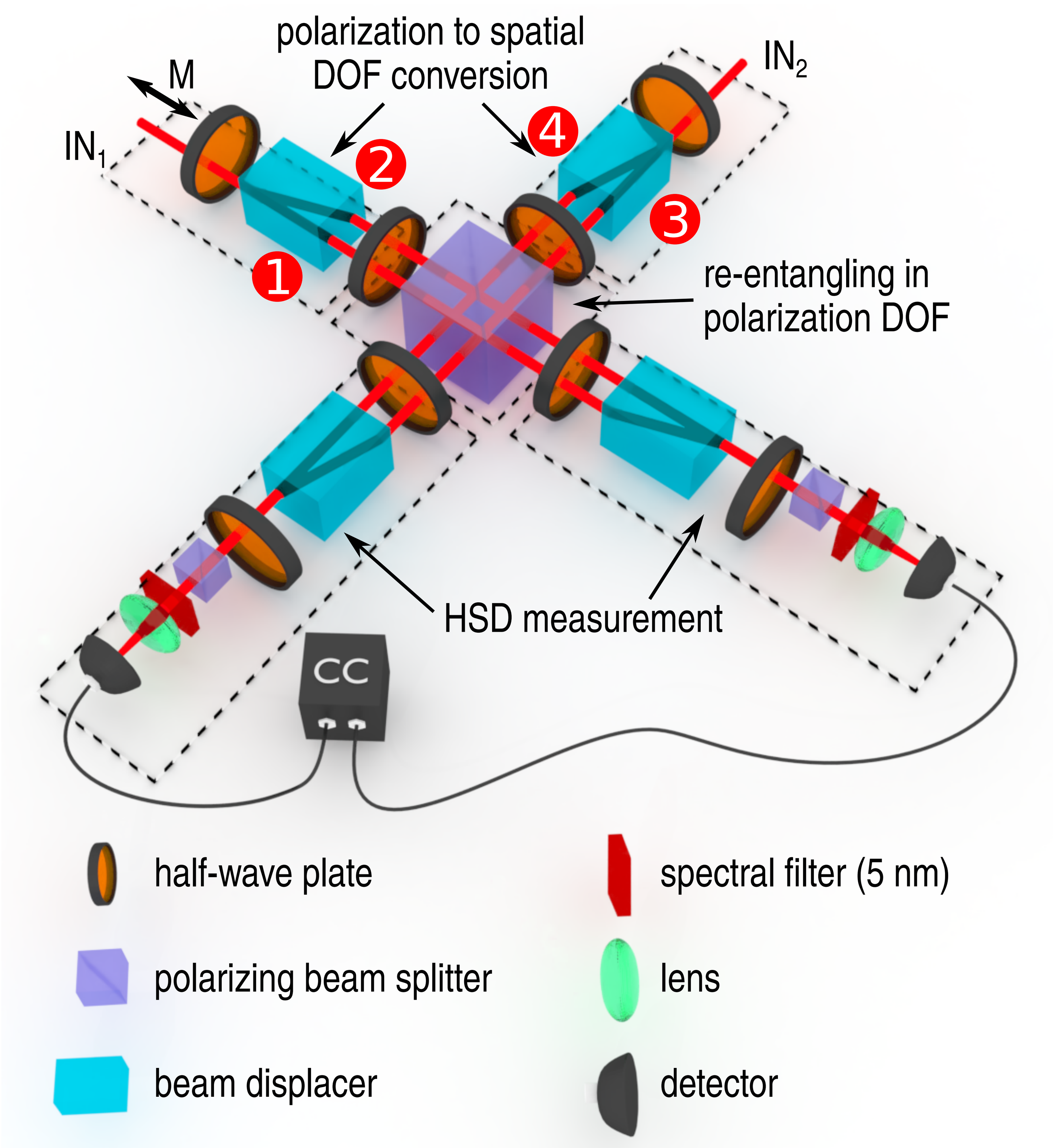
The rates and mutual phase shift between horizontally and vertically polarized photons can be tuned by adjusting the pump beam polarization and tilting one of the beam displacers. By doing so one can prepare states with various amount of entanglement. Each photon from the generated pair is coupled into a single–mode optical fiber and brought to one input port of the experimental setup.

The photons then pass through beam desplacers where the initial polarization encoding is transformed into spatial encoding. Afterwards the photons interact on the polarizing beam splitter (PBS) where the information is encoded into polarization DOF once again. As a result, two, in principle different, two-qubit states are encoded into the two DOFs.

The two states are then subjected to projective measurements accompanied by postselection. The photons are filtered by 5 nm interference filters, coupled into single–mode optical fibers and brought to single-photon detectors. Motorized translation M ensures temporal overlap of the photons on PBS.

We have measured the Hilbert-Schmidt distance between the four Bell states, four separable states, Werner states and Werner-Horodecki states. To measure the HSD between any two states (ρ_1, ρ_2) the first-order overlap defined as

$$O(\rho_1, \rho_2) = \mathrm{Tr}(\rho_1 \rho_2)$$



has to be measured in three configurations i.e. $O(\rho_1,\rho_1)$, $O(\rho_2,\rho_2)$ and $O(\rho_1,\rho_2)$.

Measurement of each first-order overlap $O(\rho_1,\rho_2)$ is split into a measurement of 25 Von-Neumann-like projective measurements on each photon across its DOFs. The projective configurations are identity ($\hat{1}$) and singlet ($\hat{S}$) projections that were implemented by suitable rotation of HWPs behind the PBS. For example the set of $\hat{1} \otimes \hat{1}$ projections consists of all combinations of local projections i.e. $|H_1H_3\rangle$, $|H_2H_3\rangle$, ... , $|V_2V_4\rangle$, while the $\hat{S} \otimes \hat{S}$ consists of projections $\frac{1}{\sqrt{2}}|H_4 - V_3\rangle$ and $\frac{1}{\sqrt{2}}|H_2 - V_1\rangle$.

The coincidence rates corresponding to each projection configuration are labeled $f_{\hat{x}\hat{y}}$, where $\hat{x}, \hat{y} \in \{\hat{1}, \hat{S}\}$. The mean value of the overlap operators relates to these rates as

$$O(\rho_1, \rho_2) = \frac{4f_{\hat{1}\hat{1}} - 8f_{\hat{S}\hat{1}} - 8f_{\hat{1}\hat{S}} + 16f_{\hat{S}\hat{S}}}{4f_{\hat{1}\hat{1}}}$$

The HSD is calculated from the overlaps as

$$H(\rho_1, \rho_2) = \sqrt{O(\rho_1, \rho_1) + O(\rho_2, \rho_2) - 2O(\rho_1, \rho_2)}$$

Results

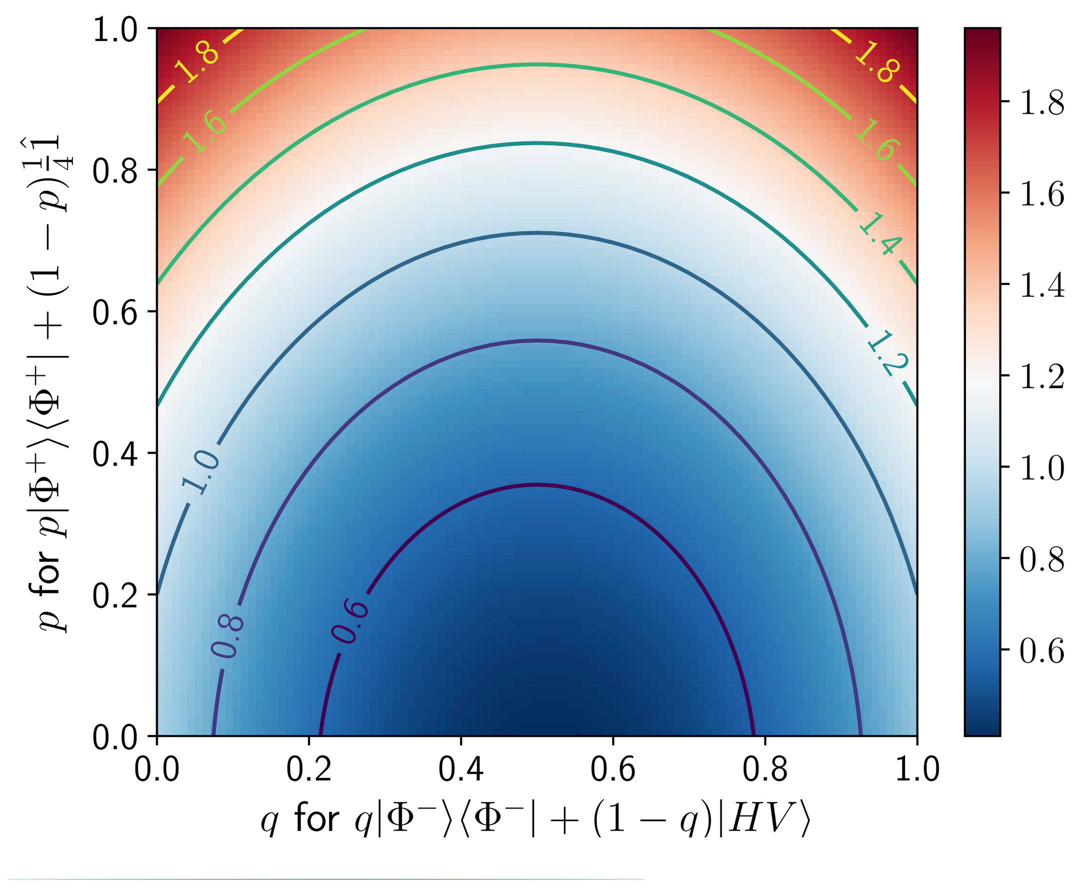
Experimental results and theoretical values (in parentheses) of the second power of Hilbert-Schmidt distance between Bell states. The vertical and horizontal axes represent polarization and spatial encoding respectively.

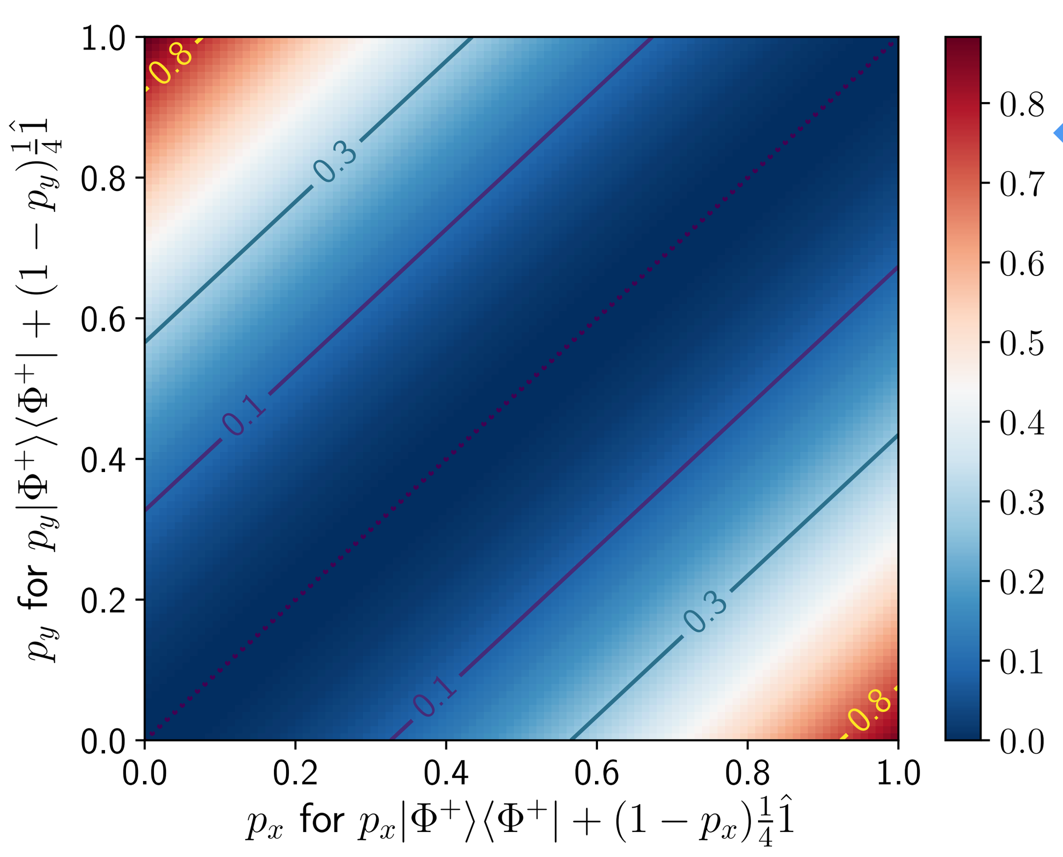
	$\vert\Phi^+\rangle$	$\vert\Phi^-\rangle$	$\vert\Psi^+\rangle$	$\vert\Psi^-\rangle$
$\vert\Phi^+\rangle$	0.10±0.07 (0.0)			
$\vert\Phi^-\rangle$	1.96±0.02 (2.0)	0.01±0.02 (0.0)		
$\vert\Psi^+\rangle$	1.74±0.02 (2.0)	1.85±0.02 (2.0)	0.03±0.05 (0.0)	
$\vert\Psi^-\rangle$	1.74±0.02 (2.0)	1.74±0.02 (2.0)	1.69±0.04 (2.0)	0.03±0.06 (0.0)

	$\vert 00\rangle$	$\vert 11\rangle$	$\vert 01\rangle$	$\vert 10\rangle$
$\vert 00\rangle$	0.001±0.002 (0.0)			
$\vert 11\rangle$	2.210±0.040 (2.0)	0.003±0.004 (0.0)		
$\vert 01\rangle$	1.850±0.020 (2.0)	1.930±0.020 (2.0)	0.001±0.003 (0.0)	
$\vert 10\rangle$	1.930±0.020 (2.0)	1.800±0.020 (2.0)	1.960±0.040 (2.0)	0.005±0.007 (0.0)

Experimental results and theoretical values (in parentheses) of the second power of Hilbert-Schmidt distance between separable states. The vertical and horizontal axes represent polarization and spatial encoding respectively.

The second power of Hilbert-Schmidt distance between Werner and Horodecki states for various weight parameters q and p. Contours represent theoretical values. The vertical and horizontal axes represent polarization and spatial encoding respectively.





The second power of Hilbert-Schmidt distance between Werner states for various weight parameters p_x and p_y. Contours represent theoretical values. The vertical and horizontal axes represent polarization and spatial encoding respectively.

[1] K. Bartkiewicz, V. Trávníček and K. Lemr, Phys. Rev. A 99, 032336 (2019).

[2] P. G. Kwiat, E. Waks, A. G. White, I. Appelbaum, and P. H. Eberhard, Phys. Rev. A 60, R773(R) (1999).

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